Chapter 8
Hypothesis Testing

8-4 Testing a Claim About a Mean: σ Known

8-5 Testing a Claim About a Mean: σ Not Known

Section 8-4
Testing a Claim About a Mean: σ Known

Notation

n = sample size

$\overline{X}$ = sample mean

$\mu_{\overline{X}}$ = population mean of all sample means from samples of size n

σ = known value of the population standard deviation

Requirements for Testing Claims About a Population Mean (with σ Known)

1) The sample is a simple random sample.

2) The value of the population standard deviation σ is known.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Test Statistic for Testing a Claim About a Mean (with σ Known)

$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}}$$

Example:

People have died in boat accidents because an obsolete estimate of the mean weight of men was used. Using the weights of the simple random sample of men from Data Set 1 in Appendix B, we obtain these sample statistics: $n = 40$ and $\overline{x} = 172.55$ lb. Research from several other sources suggests that the population of weights of men has a standard deviation given by $\sigma = 26$ lb. Use these results to test the claim that men have a mean weight greater than 166.3 lb, which was the weight in the National Transportation and Safety Board's recommendation M-04-04. Use a 0.05 significance level, and use the *P*-value method outlined in Figure 8-8.

Example:

Requirements are satisfied: simple random sample, σ is known (26 lb), sample size is 40 ($n > 30$)

Step 1: Express claim as $\mu > 166.3$ lb

Step 2: alternative to claim is $\mu \leq 166.3$ lb

Step 3: $\mu > 166.3$ lb does not contain equality, it is the alternative hypothesis:

H_0: $\mu = 166.3$ lb null hypothesis

H_1: $\mu > 166.3$ lb alternative hypothesis and original claim

Example:

Step 4: significance level is α = 0.05

Step 5: claim is about the population mean, so the relevant statistic is the sample mean (172.55 lb), σ is known (26 lb), sample size greater than 30

Step 6: calculate z

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26}{\sqrt{40}}} = 1.52$$

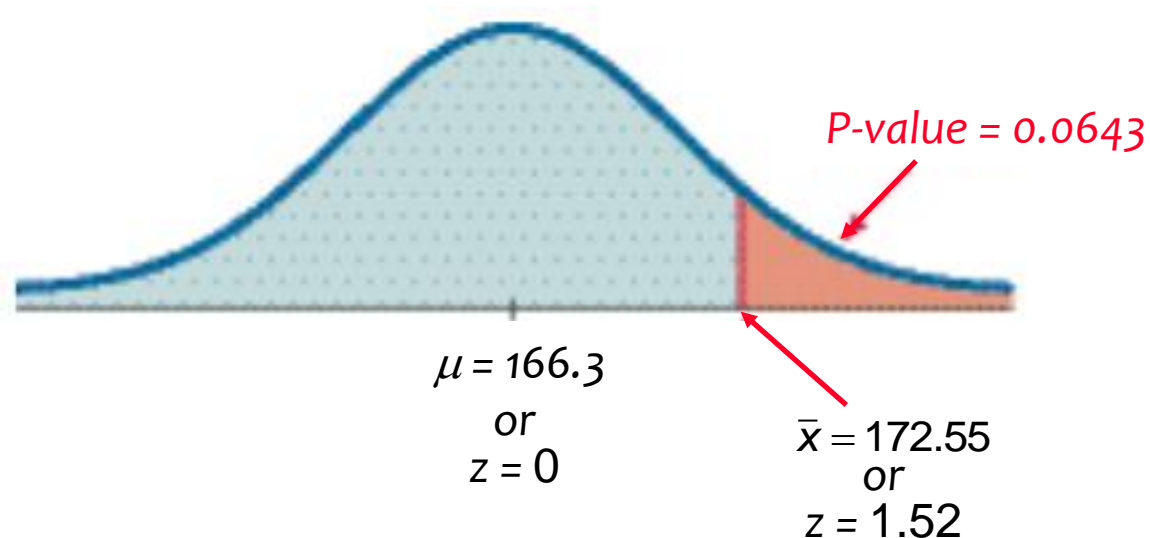**right-tailed test, so *P*-value is the area is to the right of *z* = 1.52;**

Example:

Table A-2: area to the left of $z = 1.52$ is 0.9357, so the area to the right is $1 - 0.9357 = 0.0643$.

The *P*-value is 0.0643

Step 7: The *P*-value of 0.0643 is greater than the significance level of $\alpha = 0.05$, we fail to reject the null hypothesis.

P-value = 0.0643

$\mu = 166.3$
or
$z = 0$

$\bar{x} = 172.55$
or
$z = 1.52$

Example:

The *P*-value of 0.0643 tells us that if men have a mean weight given by $\mu = 166.3$ lb, there is a good chance (0.0643) of getting a sample mean of 172.55 lb. A sample mean such as 172.55 lb could easily occur by chance. There is not sufficient evidence to support a conclusion that the population mean is greater than 166.3 lb, as in the National Transportation and Safety Board's recommendation.

Example:

The traditional method: Use $z = 1.645$ instead of finding the *P*-value. Since $z = 1.52$ does not fall in the critical region, again fail to reject the null hypothesis.

Confidence Interval method: Use a one-tailed test with a = 0.05, so construct a 90% confidence interval:

$$165.8 < \mu < 179.3$$

The confidence interval contains 166.3 lb, we cannot support a claim that μ is greater than 166.3. Again, fail to reject the null hypothesis.

Underlying Rationale of Hypothesis Testing

If, under a given assumption, there is an extremely small probability of getting sample results at least as extreme as the results that were obtained, we conclude that the assumption is probably not correct.

When testing a claim, we make an assumption (null hypothesis) of equality. We then compare the assumption and the sample results and we form one of the following conclusions:

Underlying Rationale of Hypotheses Testing - cont

- **If the sample results (or more extreme results) can easily occur when the assumption (null hypothesis) is true, we attribute the relatively small discrepancy between the assumption and the sample results to chance.**

- **If the sample results cannot easily occur when that assumption (null hypothesis) is true, we explain the relatively large discrepancy between the assumption and the sample results by concluding that the assumption is not true, so we reject the assumption.**

Recap

In this section we have discussed:

❖ **Requirements for testing claims about population means, σ known.**

❖ ***P*-value method.**

❖ **Traditional method.**

❖ **Confidence interval method.**

❖ **Rationale for hypothesis testing.**

Section 8-5
Testing a Claim About a Mean: σ Not Known

Notation

n = sample size

$\overline{X}$ = sample mean

$\mu_{\overline{X}}$ = population mean of all sample means from samples of size n

Requirements for Testing Claims About a Population Mean (with σ Not Known)

1) The sample is a simple random sample.

2) The value of the population standard deviation σ is **not** known.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Test Statistic for Testing a Claim About a Mean (with σ Not Known)

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

P-values and Critical Values

❖ **Found in Table A-3**

❖ **Degrees of freedom (df) = *n* – 1**

Choosing between the Normal and Student *t* Distributions when Testing a Claim about a Population Mean *μ*

Use the Student *t* distribution when σ is not known and either or both of these conditions is satisfied:

The population is normally distributed or *n* > 30.

Example:

People have died in boat accidents because an obsolete estimate of the mean weight of men was used. Using the weights of the simple random sample of men from Data Set 1 in Appendix B, we obtain these sample statistics: $n = 40$ and $\overline{X} = 172.55$ lb, and $\sigma = 26.33$ lb. Do not assume that the value of σ is known. Use these results to test the claim that men have a mean weight greater than 166.3 lb, which was the weight in the National Transportation and Safety Board's recommendation M-04-04. Use a 0.05 significance level, and the traditional method outlined in Figure 8-9.

Example:

Requirements are satisfied: simple random sample, population standard deviation is not known, sample size is 40 ($n > 30$)

Step 1: Express claim as $\mu > 166.3$ lb

Step 2: alternative to claim is $\mu \leq 166.3$ lb

Step 3: $\mu > 166.3$ lb does not contain equality, it is the alternative hypothesis:

H_0: $\mu = 166.3$ lb null hypothesis
H_1: $\mu > 166.3$ lb alternative hypothesis and original claim

Example:

Step 4: significance level is α = 0.05

Step 5: claim is about the population mean, so the relevant statistic is the sample mean, 172.55 lb
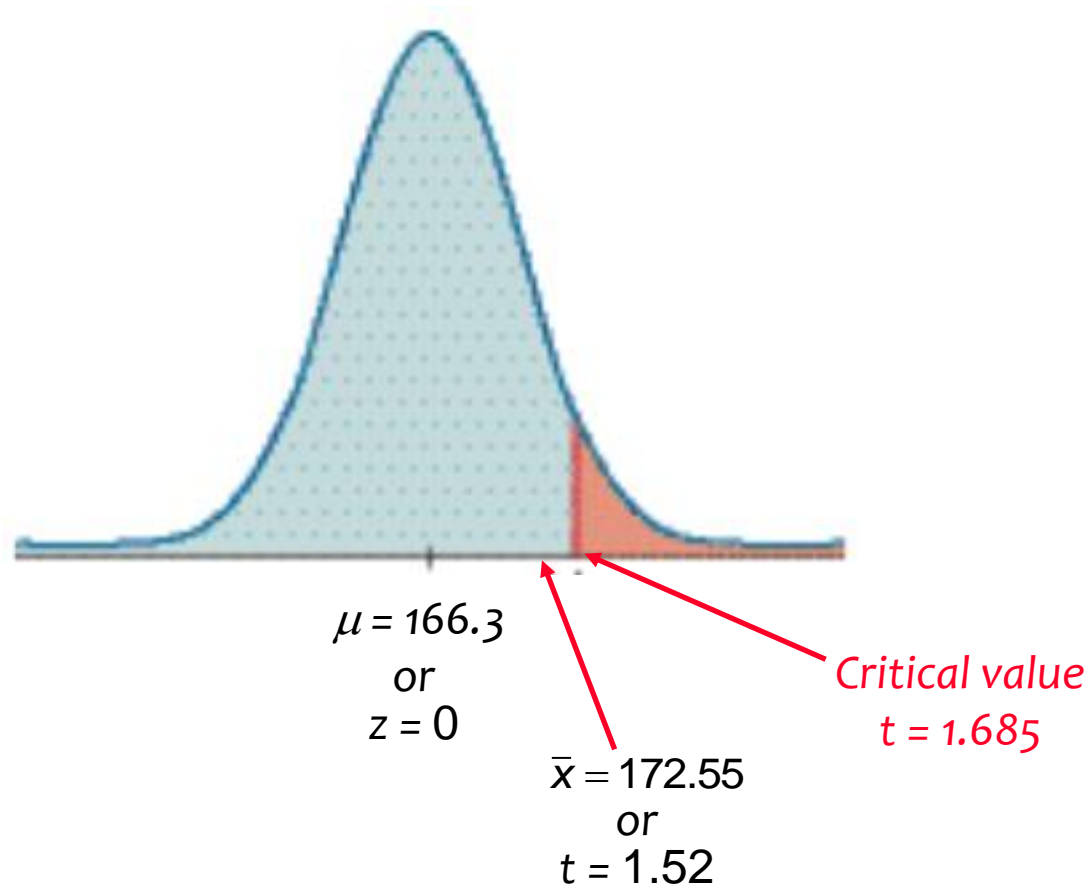
Step 6: calculate t

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26.33}{\sqrt{40}}} = 1.501$$

df = n − 1 = 39, area of 0.05, one-tail yields t = 1.685;

Example:

Step 7: **$t = 1.501$ does not fall in the critical region bounded by $t = 1.685$, we fail to reject the null hypothesis.**

$\mu = 166.3$
or
$z = 0$

$\bar{x} = 172.55$
or
$t = 1.52$

Critical value
$t = 1.685$

Example:

Because we fail to reject the null hypothesis, we conclude that there is not sufficient evidence to support a conclusion that the population mean is greater than 166.3 lb, as in the National Transportation and Safety Board's recommendation.

Normal Distribution Versus Student *t* Distribution

The critical value in the preceding example was *t* = 1.782, but if the normal distribution were being used, the critical value would have been *z* = 1.645.

The Student *t* critical value is larger (farther to the right), showing that with the Student *t* distribution, the sample evidence must be more extreme before we can consider it to be significant.

P-Value Method

❖ **Use software or a TI-83/84 Plus calculator.**

❖ **If technology is not available, use Table A-3 to identify a range of *P*-values.**

Example: Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the *P*-value corresponding to the given results.

a) In a left-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -2.007$.

b) In a right-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = 1.222$.

c) In a two-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -3.456$.

Example: Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the *P*-value corresponding to the given results.

Table A-3 Finding *P*-Values from Table A-3					
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
• • • 11	 3.106	 2.718	 2.201	 1.796	 1.363
	• •				

Example: Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the *P*-value corresponding to the given results.

a) The test is a left-tailed test with test statistic *t* = –2.007, so the *P*-value is the area to the left of –2.007. Because of the symmetry of the *t* distribution, that is the same as the area to the right of +2.007. Any test statistic between 2.201 and 1.796 has a right-tailed *P*-value that is between 0.025 and 0.05. We conclude that

$$0.025 < P\text{-value} < 0.05.$$

Example: Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the *P*-value corresponding to the given results.

b) The test is a right-tailed test with test statistic *t* = 1.222, so the *P*-value is the area to the right of 1.222. Any test statistic less than 1.363 has a right-tailed *P*-value that is greater than 0.10. We conclude that *P*-value > 0.10.

Example: Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the *P*-value corresponding to the given results.

c) The test is a two-tailed test with test statistic $t = -3.456$. The *P*-value is twice the area to the right of -3.456. Any test statistic greater than 3.106 has a two-tailed *P*-value that is less than 0.01. We conclude that *P*-value < 0.01.

Recap

In this section we have discussed:

❖ **Assumptions for testing claims about population means, *σ* unknown.**

❖ **Student *t* distribution.**

❖ ***P*-value method.**